1. $f(x) = x - 3 = x^1 - 3 \quad \Rightarrow \quad F(x) = \dfrac{x^{1+1}}{1+1} - 3x + C = \frac{1}{2}x^2 - 3x + C$

Check: $F'(x) = \frac{1}{2}(2x) - 3 + 0 = x - 3 = f(x)$

2. $f(x) = \frac{1}{2}x^2 - 2x + 6 \quad \Rightarrow \quad F(x) = \dfrac{1}{2}\dfrac{x^3}{3} - 2\dfrac{x^2}{2} + 6x + C = \frac{1}{6}x^3 - x^2 + 6x + C$

3. $f(x) = \frac{1}{2} + \frac{3}{4}x^2 - \frac{4}{5}x^3 \quad \Rightarrow \quad F(x) = \frac{1}{2}x + \dfrac{3}{4}\dfrac{x^{2+1}}{2+1} - \dfrac{4}{5}\dfrac{x^{3+1}}{3+1} + C = \frac{1}{2}x + \frac{1}{4}x^3 - \frac{1}{5}x^4 + C$

Check: $F'(x) = \frac{1}{2} + \frac{1}{4}(3x^2) - \frac{1}{5}(4x^3) + 0 = \frac{1}{2} + \frac{3}{4}x^2 - \frac{4}{5}x^3 = f(x)$

4. $f(x) = 8x^9 - 3x^6 + 12x^3 \quad \Rightarrow \quad F(x) = 8\left(\frac{1}{10}x^{10}\right) - 3\left(\frac{1}{7}x^7\right) + 12\left(\frac{1}{4}x^4\right) + C = \frac{4}{5}x^{10} - \frac{3}{7}x^7 + 3x^4 + C$

5. $f(x) = (x+1)(2x-1) = 2x^2 + x - 1 \quad \Rightarrow \quad F(x) = 2\left(\frac{1}{3}x^3\right) + \frac{1}{2}x^2 - x + C = \frac{2}{3}x^3 + \frac{1}{2}x^2 - x + C$

6. $f(x) = x(2-x)^2 = x\left(4 - 4x + x^2\right) = 4x - 4x^2 + x^3 \quad \Rightarrow$

$F(x) = 4\left(\frac{1}{2}x^2\right) - 4\left(\frac{1}{3}x^3\right) + \frac{1}{4}x^4 + C = 2x^2 - \frac{4}{3}x^3 + \frac{1}{4}x^4 + C$

7. $f(x) = 5x^{1/4} - 7x^{3/4} \quad \Rightarrow \quad F(x) = 5\dfrac{x^{1/4+1}}{\frac{1}{4}+1} - 7\dfrac{x^{3/4+1}}{\frac{3}{4}+1} + C = 5\dfrac{x^{5/4}}{5/4} - 7\dfrac{x^{7/4}}{7/4} + C = 4x^{5/4} - 4x^{7/4} + C$

8. $f(x) = 2x + 3x^{1.7} \quad \Rightarrow \quad F(x) = x^2 + \frac{3}{2.7}x^{2.7} + C = x^2 + \frac{10}{9}x^{2.7} + C$

9. $f(x) = 6\sqrt{x} - \sqrt[6]{x} = 6x^{1/2} - x^{1/6} \quad \Rightarrow$

$F(x) = 6\dfrac{x^{1/2+1}}{\frac{1}{2}+1} - \dfrac{x^{1/6+1}}{\frac{1}{6}+1} + C = 6\dfrac{x^{3/2}}{3/2} - \dfrac{x^{7/6}}{7/6} + C = 4x^{3/2} - \frac{6}{7}x^{7/6} + C$

10. $f(x) = \sqrt[4]{x^3} + \sqrt[3]{x^4} = x^{3/4} + x^{4/3} \quad \Rightarrow \quad F(x) = \dfrac{x^{7/4}}{7/4} + \dfrac{x^{7/3}}{7/3} + C = \frac{4}{7}x^{7/4} + \frac{3}{7}x^{7/3} + C$

11. $f(x) = \dfrac{10}{x^9} = 10x^{-9}$ has domain $(-\infty, 0) \cup (0, \infty)$, so $F(x) = \begin{cases} \dfrac{10x^{-8}}{-8} + C_1 = -\dfrac{5}{4x^8} + C_1 & \text{if } x < 0 \\ -\dfrac{5}{4x^8} + C_2 & \text{if } x > 0 \end{cases}$

See Example 1(b) for a similar problem.

12. $g(x) = \dfrac{5 - 4x^3 + 2x^6}{x^6} = 5x^{-6} - 4x^{-3} + 2$ has domain $(-\infty, 0) \cup (0, \infty)$, so

$$G(x) = \begin{cases} 5\dfrac{x^{-5}}{-5} - 4\dfrac{x^{-2}}{-2} + 2x + C_1 = -\dfrac{1}{x^5} + \dfrac{2}{x^2} + 2x + C_1 & \text{if } x < 0 \\ -\dfrac{1}{x^5} + \dfrac{2}{x^2} + 2x + C_2 & \text{if } x > 0 \end{cases}$$

13. $f(u) = \dfrac{u^4 + 3\sqrt{u}}{u^2} = \dfrac{u^4}{u^2} + \dfrac{3u^{1/2}}{u^2} = u^2 + 3u^{-3/2} \Rightarrow$

$$F(u) = \frac{u^3}{3} + 3\frac{u^{-3/2+1}}{-3/2+1} + C = \frac{1}{3}u^3 + 3\frac{u^{-1/2}}{-1/2} + C = \frac{1}{3}u^3 - \frac{6}{\sqrt{u}} + C$$

14. $f(x) = 3e^x + 7\sec^2 x \Rightarrow F(x) = 3e^x + 7\tan x + C_n$ on the interval $\left(n\pi - \frac{\pi}{2}, n\pi + \frac{\pi}{2}\right)$.

15. $g(\theta) = \cos\theta - 5\sin\theta \Rightarrow G(\theta) = \sin\theta - 5(-\cos\theta) + C = \sin\theta + 5\cos\theta + C$

16. $f(t) = \sin t + 2\sinh t \Rightarrow F(t) = -\cos t + 2\cosh t + C$

17. $f(x) = 5e^x - 3\cosh x \Rightarrow F(x) = 5e^x - 3\sinh x + C$

18. $f(x) = 2\sqrt{x} + 6\cos x = 2x^{1/2} + 6\cos x \Rightarrow F(x) = 2\left(\dfrac{x^{3/2}}{3/2}\right) + 6\sin x + C = \frac{4}{3}x^{3/2} + 6\sin x + C$

19. $f(x) = \dfrac{x^5 - x^3 + 2x}{x^4} = x - \dfrac{1}{x} + \dfrac{2}{x^3} = x - \dfrac{1}{x} + 2x^{-3} \Rightarrow$

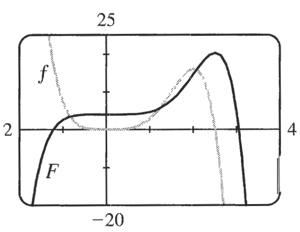
$$F(x) = \frac{x^2}{2} - \ln|x| + 2\left(\frac{x^{-3+1}}{-3+1}\right) + C = \tfrac{1}{2}x^2 - \ln|x| - \frac{1}{x^2} + C$$

20. $f(x) = \dfrac{2 + x^2}{1 + x^2} = \dfrac{1 + (1 + x^2)}{1 + x^2} = \dfrac{1}{1 + x^2} + 1 \Rightarrow F(x) = \tan^{-1} x + x + C$

21. $f(x) = 5x^4 - 2x^5 \Rightarrow F(x) = 5 \cdot \dfrac{x^5}{5} - 2 \cdot \dfrac{x^6}{6} + C = x^5 - \frac{1}{3}x^6 + C$.

$F(0) = 4 \Rightarrow 0^5 - \frac{1}{3} \cdot 0^6 + C = 4 \Rightarrow C = 4$, so $F(x) = x^5 - \frac{1}{3}x^6 + 4$.

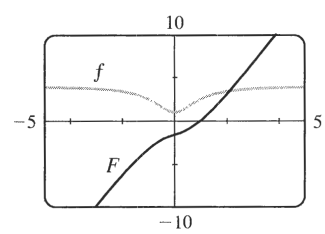
The graph confirms our answer since $f(x) = 0$ when F has a local maximum, f is positive when F is increasing, and f is negative when F is decreasing.



22. $f(x) = 4 - 3(1 + x^2)^{-1} = 4 - \dfrac{3}{1 + x^2} \Rightarrow F(x) = 4x - 3\tan^{-1} x + C$.

$F(1) = 0 \Rightarrow 4 - 3\left(\frac{\pi}{4}\right) + C = 0 \Rightarrow C = \frac{3\pi}{4} - 4$, so

$F(x) = 4x - 3\tan^{-1} x + \frac{3\pi}{4} - 4$. Note that f is positive and F is increasing on $\mathbb{R}$. Also, f has smaller values where the slopes of the tangent lines of F are smaller.



23. $f''(x) = 6x + 12x^2 \Rightarrow f'(x) = 6 \cdot \dfrac{x^2}{2} + 12 \cdot \dfrac{x^3}{3} + C = 3x^2 + 4x^3 + C \Rightarrow$

$f(x) = 3 \cdot \dfrac{x^3}{3} + 4 \cdot \dfrac{x^4}{4} + Cx + D = x^3 + x^4 + Cx + D$ [C and D are just arbitrary constants]

24. $f''(x) = 2 + x^3 + x^6 \Rightarrow f'(x) = 2x + \frac{1}{4}x^4 + \frac{1}{7}x^7 + C \Rightarrow f(x) = x^2 + \frac{1}{20}x^5 + \frac{1}{56}x^8 + Cx + D$

25. $f''(x) = \frac{2}{3}x^{2/3} \Rightarrow f'(x) = \frac{2}{3}\left(\frac{x^{5/3}}{5/3}\right) + C = \frac{2}{5}x^{5/3} + C \Rightarrow f(x) = \frac{2}{5}\left(\frac{x^{8/3}}{8/3}\right) + Cx + D = \frac{3}{20}x^{8/3} + Cx + D$

26. $f''(x) = 6x + \sin x \Rightarrow f'(x) = 6\left(\frac{x^2}{2}\right) - \cos x + C = 3x^2 - \cos x + C \Rightarrow$

$f(x) = 3\left(\frac{x^3}{3}\right) - \sin x + Cx + D = x^3 - \sin x + Cx + D$

27. $f'''(t) = e^t \Rightarrow f''(t) = e^t + C \Rightarrow f'(t) = e^t + Ct + D \Rightarrow f(t) = e^t + \frac{1}{2}Ct^2 + Dt + E$

28. $f'''(t) = t - \sqrt{t} \Rightarrow f''(t) = \frac{1}{2}t^2 - \frac{2}{3}t^{3/2} + C \Rightarrow f'(t) = \frac{1}{6}t^3 - \frac{4}{15}t^{5/2} + Ct + D \Rightarrow$

$f(t) = \frac{1}{24}t^4 - \frac{8}{105}t^{7/2} + \frac{1}{2}Ct^2 + Dt + E$

29. $f'(x) = 1 - 6x \Rightarrow f(x) = x - 3x^2 + C$. $f(0) = C$ and $f(0) = 8 \Rightarrow C = 8$, so $f(x) = x - 3x^2 + 8$.

30. $f'(x) = 8x^3 + 12x + 3 \Rightarrow f(x) = 2x^4 + 6x^2 + 3x + C$. $f(1) = 11 + C$ and $f(1) = 6 \Rightarrow$

$11 + C = 6 \Rightarrow C = -5$, so $f(x) = 2x^4 + 6x^2 + 3x - 5$.

31. $f'(x) = \sqrt{x}(6 + 5x) = 6x^{1/2} + 5x^{3/2} \Rightarrow f(x) = 4x^{3/2} + 2x^{5/2} + C$.

$f(1) = 6 + C$ and $f(1) = 10 \Rightarrow C = 4$, so $f(x) = 4x^{3/2} + 2x^{5/2} + 4$.

32. $f'(x) = 2x - 3/x^4 = 2x - 3x^{-4} \Rightarrow f(x) = x^2 + x^{-3} + C$ because we're given that $x > 0$.

$f(1) = 2 + C$ and $f(1) = 3 \Rightarrow C = 1$, so $f(x) = x^2 + 1/x^3 + 1$.

33. $f'(t) = 2\cos t + \sec^2 t \Rightarrow f(t) = 2\sin t + \tan t + C$ because $-\pi/2 < t < \pi/2$.

$f\left(\frac{\pi}{3}\right) = 2\left(\sqrt{3}/2\right) + \sqrt{3} + C = 2\sqrt{3} + C$ and $f\left(\frac{\pi}{3}\right) = 4 \Rightarrow C = 4 - 2\sqrt{3}$, so $f(t) = 2\sin t + \tan t + 4 - 2\sqrt{3}$.

34. $f'(x) = \frac{x^2 - 1}{x} = x - \frac{1}{x}$ has domain $(-\infty, 0) \cup (0, \infty) \Rightarrow f(x) = \begin{cases} \frac{1}{2}x^2 - \ln x + C_1 & \text{if } x > 0 \\ \frac{1}{2}x^2 - \ln(-x) + C_2 & \text{if } x < 0 \end{cases}$

$f(1) = \frac{1}{2} - \ln 1 + C_1 = \frac{1}{2} + C_1$ and $f(1) = \frac{1}{2} \Rightarrow C_1 = 0$.

$f(-1) = \frac{1}{2} - \ln 1 + C_2 = \frac{1}{2} + C_2$ and $f(-1) = 0 \Rightarrow C_2 = -\frac{1}{2}$.

Thus, $f(x) = \begin{cases} \frac{1}{2}x^2 - \ln x & \text{if } x > 0 \\ \frac{1}{2}x^2 - \ln(-x) - \frac{1}{2} & \text{if } x < 0 \end{cases}$

35. $f'(x) = x^{-1/3}$ has domain $(-\infty, 0) \cup (0, \infty) \Rightarrow f(x) = \begin{cases} \frac{3}{2}x^{2/3} + C_1 & \text{if } x > 0 \\ \frac{3}{2}x^{2/3} + C_2 & \text{if } x < 0 \end{cases}$

$f(1) = \frac{3}{2} + C_1$ and $f(1) = 1 \Rightarrow C_1 = -\frac{1}{2}$. $f(-1) = \frac{3}{2} + C_2$ and $f(-1) = -1 \Rightarrow C_2 = -\frac{5}{2}$.

Thus, $f(x) = \begin{cases} \frac{3}{2}x^{2/3} - \frac{1}{2} & \text{if } x > 0 \\ \frac{3}{2}x^{2/3} - \frac{5}{2} & \text{if } x < 0 \end{cases}$

36. $f'(x) = 4/\sqrt{1 - x^2} \Rightarrow f(x) = 4\sin^{-1} x + C$. $f\left(\frac{1}{2}\right) = 4\sin^{-1}\left(\frac{1}{2}\right) + C = 4 \cdot \frac{\pi}{6} + C$ and $f\left(\frac{1}{2}\right) = 1 \Rightarrow$

$\frac{2\pi}{3} + C = 1 \Rightarrow C = 1 - \frac{2\pi}{3}$, so $f(x) = 4\sin^{-1} x + 1 - \frac{2\pi}{3}$.

37. $f''(x) = 24x^2 + 2x + 10 \;\Rightarrow\; f'(x) = 8x^3 + x^2 + 10x + C$. $f'(1) = 8 + 1 + 10 + C$ and $f'(1) = -3 \;\Rightarrow$ $19 + C = -3 \;\Rightarrow\; C = -22$, so $f'(x) = 8x^3 + x^2 + 10x - 22$ and hence, $f(x) = 2x^4 + \frac{1}{3}x^3 + 5x^2 - 22x + D$. $f(1) = 2 + \frac{1}{3} + 5 - 22 + D$ and $f(1) = 5 \;\Rightarrow\; D = 22 - \frac{7}{3} = \frac{59}{3}$, so $f(x) = 2x^4 + \frac{1}{3}x^3 + 5x^2 - 22x + \frac{59}{3}$.

38. $f''(x) = 4 - 6x - 40x^3 \;\Rightarrow\; f'(x) = 4x - 3x^2 - 10x^4 + C$. $f'(0) = C$ and $f'(0) = 1 \;\Rightarrow\; C = 1$, so $f'(x) = 4x - 3x^2 - 10x^4 + 1$ and hence, $f(x) = 2x^2 - x^3 - 2x^5 + x + D$. $f(0) = D$ and $f(0) = 2 \;\Rightarrow\; D = 2$, so $f(x) = 2x^2 - x^3 - 2x^5 + x + 2$.

39. $f''(\theta) = \sin\theta + \cos\theta \;\Rightarrow\; f'(\theta) = -\cos\theta + \sin\theta + C$. $f'(0) = -1 + C$ and $f'(0) = 4 \;\Rightarrow\; C = 5$, so $f'(\theta) = -\cos\theta + \sin\theta + 5$ and hence, $f(\theta) = -\sin\theta - \cos\theta + 5\theta + D$. $f(0) = -1 + D$ and $f(0) = 3 \;\Rightarrow\; D = 4$, so $f(\theta) = -\sin\theta - \cos\theta + 5\theta + 4$.

40. $f''(t) = 3/\sqrt{t} = 3t^{-1/2} \;\Rightarrow\; f'(t) = 6t^{1/2} + C$. $f'(4) = 12 + C$ and $f'(4) = 7 \;\Rightarrow\; C = -5$, so $f'(t) = 6t^{1/2} - 5$ and hence, $f(t) = 4t^{3/2} - 5t + D$. $f(4) = 32 - 20 + D$ and $f(4) = 20 \;\Rightarrow\; D = 8$, so $f(t) = 4t^{3/2} - 5t + 8$.

41. $f''(x) = 2 - 12x \;\Rightarrow\; f'(x) = 2x - 6x^2 + C \;\Rightarrow\; f(x) = x^2 - 2x^3 + Cx + D$. $f(0) = D$ and $f(0) = 9 \;\Rightarrow\; D = 9$. $f(2) = 4 - 16 + 2C + 9 = 2C - 3$ and $f(2) = 15 \;\Rightarrow\; 2C = 18 \;\Rightarrow$ $C = 9$, so $f(x) = x^2 - 2x^3 + 9x + 9$.

42. $f''(x) = 20x^3 + 12x^2 + 4 \;\Rightarrow\; f'(x) = 5x^4 + 4x^3 + 4x + C \;\Rightarrow\; f(x) = x^5 + x^4 + 2x^2 + Cx + D$. $f(0) = D$ and $f(0) = 8 \;\Rightarrow\; D = 8$. $f(1) = 1 + 1 + 2 + C + 8 = C + 12$ and $f(1) = 5 \;\Rightarrow\; C = -7$, so $f(x) = x^5 + x^4 + 2x^2 - 7x + 8$.

43. $f''(x) = 2 + \cos x \;\Rightarrow\; f'(x) = 2x + \sin x + C \;\Rightarrow\; f(x) = x^2 - \cos x + Cx + D$. $f(0) = -1 + D$ and $f(0) = -1 \;\Rightarrow\; D = 0$. $f\left(\frac{\pi}{2}\right) = \pi^2/4 + \left(\frac{\pi}{2}\right)C$ and $f\left(\frac{\pi}{2}\right) = 0 \;\Rightarrow\; \left(\frac{\pi}{2}\right)C = -\pi^2/4 \;\Rightarrow$ $C = -\frac{\pi}{2}$, so $f(x) = x^2 - \cos x - \left(\frac{\pi}{2}\right)x$.

44. $f''(t) = 2e^t + 3\sin t \;\Rightarrow\; f'(t) = 2e^t - 3\cos t + C \;\Rightarrow\; f(t) = 2e^t - 3\sin t + Ct + D$. $f(0) = 2 + D$ and $f(0) = 0 \;\Rightarrow\; D = -2$. $f(\pi) = 2e^\pi + \pi C - 2$ and $f(\pi) = 0 \;\Rightarrow\; \pi C = 2 - 2e^\pi \;\Rightarrow\; C = \dfrac{2 - 2e^\pi}{\pi}$, so $f(t) = 2e^t - 3\sin t + \dfrac{2 - 2e^\pi}{\pi}t - 2$.

45. $f''(x) = x^{-2},\ x > 0 \;\Rightarrow\; f'(x) = -1/x + C \;\Rightarrow\; f(x) = -\ln|x| + Cx + D = -\ln x + Cx + D$ [since $x > 0$]. $f(1) = 0 \;\Rightarrow\; C + D = 0$ and $f(2) = 0 \;\Rightarrow\; -\ln 2 + 2C + D = 0 \;\Rightarrow\; -\ln 2 + 2C - C = 0$ [since $D = -C$] $\Rightarrow$ $-\ln 2 + C = 0 \;\Rightarrow\; C = \ln 2$ and $D = -\ln 2$. So $f(x) = -\ln x + (\ln 2)x - \ln 2$.

46. $f'''(x) = \cos x \;\Rightarrow\; f''(x) = \sin x + C$. $f''(0) = C$ and $f''(0) = 3 \;\Rightarrow\; C = 3$. $f''(x) = \sin x + 3 \;\Rightarrow$ $f'(x) = -\cos x + 3x + D$. $f'(0) = -1 + D$ and $f'(0) = 2 \;\Rightarrow\; D = 3$. $f'(x) = -\cos x + 3x + 3 \;\Rightarrow$ $f(x) = -\sin x + \frac{3}{2}x^2 + 3x + E$. $f(0) = E$ and $f(0) = 1 \;\Rightarrow\; E = 1$. Thus, $f(x) = -\sin x + \frac{3}{2}x^2 + 3x + 1$.

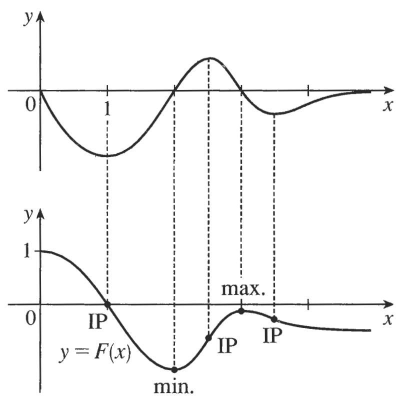
47. Given $f'(x) = 2x + 1$, we have $f(x) = x^2 + x + C$. Since f passes through $(1, 6)$, $f(1) = 6 \;\Rightarrow\; 1^2 + 1 + C = 6 \;\Rightarrow$ $C = 4$. Therefore, $f(x) = x^2 + x + 4$ and $f(2) = 2^2 + 2 + 4 = 10$.

48. $f'(x) = x^3 \;\Rightarrow\; f(x) = \frac{1}{4}x^4 + C$. $x + y = 0 \;\Rightarrow\; y = -x \;\Rightarrow\; m = -1$. Now $m = f'(x) \;\Rightarrow\; -1 = x^3 \;\Rightarrow$ $x = -1 \;\Rightarrow\; y = 1$ (from the equation of the tangent line), so $(-1, 1)$ is a point on the graph of f. From f, $1 = \frac{1}{4}(-1)^4 + C \;\Rightarrow\; C = \frac{3}{4}$. Therefore, the function is $f(x) = \frac{1}{4}x^4 + \frac{3}{4}$.

49. b is the antiderivative of f. For small x, f is negative, so the graph of its antiderivative must be decreasing. But both a and c are increasing for small x, so only b can be f's antiderivative. Also, f is positive where b is increasing, which supports our conclusion.

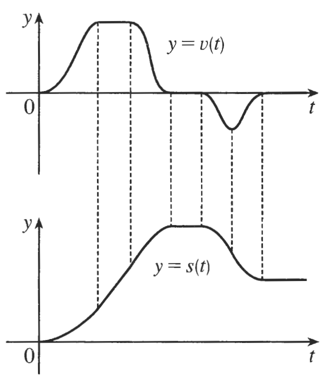
50. We know right away that c cannot be f's antiderivative, since the slope of c is not zero at the x-value where $f = 0$. Now f is positive when a is increasing and negative when a is decreasing, so a is the antiderivative of f.

51.



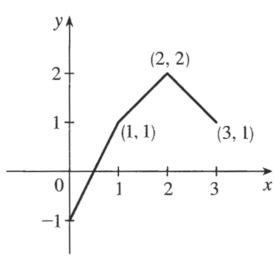
The graph of F must start at $(0, 1)$. Where the given graph, $y = f(x)$, has a local minimum or maximum, the graph of F will have an inflection point.
Where f is negative (positive), F is decreasing (increasing).
Where f changes from negative to positive, F will have a minimum.
Where f changes from positive to negative, F will have a maximum.
Where f is decreasing (increasing), F is concave downward (upward).

52.



Where v is positive (negative), s is increasing (decreasing).
Where v is increasing (decreasing), s is concave upward (downward).
Where v is horizontal (a steady velocity), s is linear.

53.



$$f'(x) = \begin{cases} 2 & \text{if } 0 \le x < 1 \\ 1 & \text{if } 1 < x < 2 \\ -1 & \text{if } 2 < x \le 3 \end{cases} \;\Rightarrow\; f(x) = \begin{cases} 2x + C & \text{if } 0 \le x < 1 \\ x + D & \text{if } 1 < x < 2 \\ -x + E & \text{if } 2 < x \le 3 \end{cases}$$

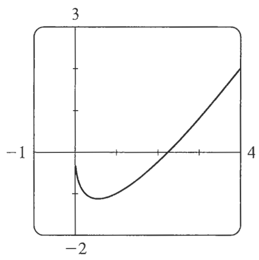
$f(0) = -1 \;\Rightarrow\; 2(0) + C = -1 \;\Rightarrow\; C = -1$. Starting at the point $(0, -1)$ and moving to the right on a line with slope 2 gets us to the point $(1, 1)$. The slope for $1 < x < 2$ is 1, so we get to the point $(2, 2)$. Here we have used the fact that f is continuous. We can include the point $x = 1$ on either the first or the second part of f. The line connecting $(1, 1)$ to $(2, 2)$ is $y = x$, so $D = 0$. The slope for

$2 < x \le 3$ is -1, so we get to $(3, 1)$. $f(3) = 1 \;\Rightarrow\; -3 + E = 1 \;\Rightarrow\; E = 4$. Thus

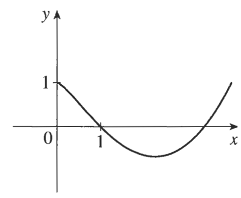
$$f(x) = \begin{cases} 2x - 1 & \text{if } 0 \le x \le 1 \\ x & \text{if } 1 < x < 2 \\ -x + 4 & \text{if } 2 \le x \le 3 \end{cases}$$

Note that $f'(x)$ does not exist at $x = 1$ or at $x = 2$.

54. (a)



(b) Since $F(0) = 1$, we can start our graph at $(0, 1)$. f has a minimum at about $x = 0.5$, so its derivative is zero there. f is decreasing on $(0, 0.5)$, so its derivative is negative and hence, F is CD on $(0, 0.5)$ and has an IP at $x \approx 0.5$. On $(0.5, 2.2)$, f is negative and increasing (f' is positive), so F is decreasing and CU. On $(2.2, \infty)$, f is positive and increasing, so F is increasing and CU.



(c) $f(x) = 2x - 3\sqrt{x} \;\Rightarrow$

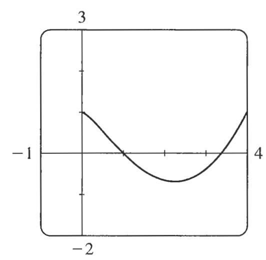
$F(x) = x^2 - 3 \cdot \frac{2}{3} x^{3/2} + C$.

$F(0) = C$ and $F(0) = 1 \;\Rightarrow$

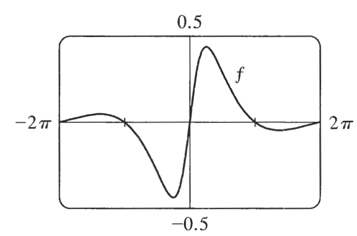
$C = 1$, so

$F(x) = x^2 - 2x^{3/2} + 1$.

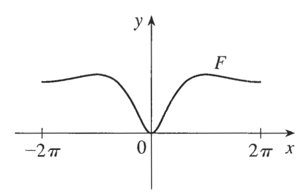
(d)



55. $f(x) = \dfrac{\sin x}{1 + x^2}$, $-2\pi \le x \le 2\pi$

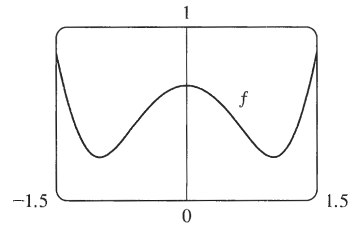


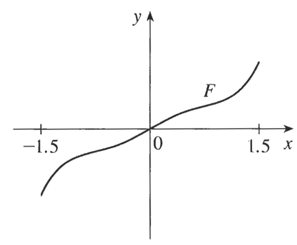
Note that the graph of f is one of an odd function, so the graph of F will be one of an even function.



56. $f(x) = \sqrt{x^4 - 2x^2 + 2} - 1, \ -1.5 \le x \le 1.5$

Note that the graph of f is one of an even function, so the graph of F will be one of an odd function.





57. $v(t) = s'(t) = \sin t - \cos t \ \Rightarrow \ s(t) = -\cos t - \sin t + C$. $s(0) = -1 + C$ and $s(0) = 0 \ \Rightarrow \ C = 1$, so $s(t) = -\cos t - \sin t + 1$.

58. $v(t) = s'(t) = 1.5\sqrt{t} \ \Rightarrow \ s(t) = t^{3/2} + C$. $s(4) = 8 + C$ and $s(4) = 10 \ \Rightarrow \ C = 2$, so $s(t) = t^{3/2} + 2$.

59. $a(t) = v'(t) = t - 2 \ \Rightarrow \ v(t) = \frac{1}{2}t^2 - 2t + C$. $v(0) = C$ and $v(0) = 3 \ \Rightarrow \ C = 3$, so $v(t) = \frac{1}{2}t^2 - 2t + 3$ and $s(t) = \frac{1}{6}t^3 - t^2 + 3t + D$. $s(0) = D$ and $s(0) = 1 \ \Rightarrow \ D = 1$, and $s(t) = \frac{1}{6}t^3 - t^2 + 3t + 1$.

60. $a(t) = v'(t) = \cos t + \sin t \ \Rightarrow \ v(t) = \sin t - \cos t + C \ \Rightarrow \ 5 = v(0) = -1 + C \ \Rightarrow \ C = 6$, so $v(t) = \sin t - \cos t + 6 \ \Rightarrow \ s(t) = -\cos t - \sin t + 6t + D \ \Rightarrow \ 0 = s(0) = -1 + D \ \Rightarrow \ D = 1$, so $s(t) = -\cos t - \sin t + 6t + 1$.

61. $a(t) = v'(t) = 10\sin t + 3\cos t \ \Rightarrow \ v(t) = -10\cos t + 3\sin t + C \ \Rightarrow \ s(t) = -10\sin t - 3\cos t + Ct + D$. $s(0) = -3 + D = 0$ and $s(2\pi) = -3 + 2\pi C + D = 12 \ \Rightarrow \ D = 3$ and $C = \frac{6}{\pi}$. Thus, $s(t) = -10\sin t - 3\cos t + \frac{6}{\pi}t + 3$.

62. $a(t) = t^2 - 4t + 6 \ \Rightarrow \ v(t) = \frac{1}{3}t^3 - 2t^2 + 6t + C \ \Rightarrow \ s(t) = \frac{1}{12}t^4 - \frac{2}{3}t^3 + 3t^2 + Ct + D$. $s(0) = D$ and $s(0) = 0 \ \Rightarrow \ D = 0$. $s(1) = \frac{29}{12} + C$ and $s(1) = 20 \ \Rightarrow \ C = \frac{211}{12}$. Thus, $s(t) = \frac{1}{12}t^4 - \frac{2}{3}t^3 + 3t^2 + \frac{211}{12}t$.

63. (a) We first observe that since the stone is dropped 450 m above the ground, $v(0) = 0$ and $s(0) = 450$. $v'(t) = a(t) = -9.8 \ \Rightarrow \ v(t) = -9.8t + C$. Now $v(0) = 0 \ \Rightarrow \ C = 0$, so $v(t) = -9.8t \ \Rightarrow$ $s(t) = -4.9t^2 + D$. Last, $s(0) = 450 \ \Rightarrow \ D = 450 \ \Rightarrow \ s(t) = 450 - 4.9t^2$.

(b) The stone reaches the ground when $s(t) = 0$. $450 - 4.9t^2 = 0 \ \Rightarrow \ t^2 = 450/4.9 \ \Rightarrow \ t_1 = \sqrt{450/4.9} \approx 9.58$ s.

(c) The velocity with which the stone strikes the ground is $v(t_1) = -9.8\sqrt{450/4.9} \approx -93.9$ m/s.

(d) This is just reworking parts (a) and (b) with $v(0) = -5$. Using $v(t) = -9.8t + C$, $v(0) = -5 \ \Rightarrow \ 0 + C = -5 \ \Rightarrow$ $v(t) = -9.8t - 5$. So $s(t) = -4.9t^2 - 5t + D$ and $s(0) = 450 \ \Rightarrow \ D = 450 \ \Rightarrow \ s(t) = -4.9t^2 - 5t + 450$. Solving $s(t) = 0$ by using the quadratic formula gives us $t = \left(5 \pm \sqrt{8845}\right)/(-9.8) \ \Rightarrow \ t_1 \approx 9.09$ s.

64. $v'(t) = a(t) = a \;\Rightarrow\; v(t) = at + C$ and $v_0 = v(0) = C \;\Rightarrow\; v(t) = at + v_0 \;\Rightarrow$

$s(t) = \frac{1}{2}at^2 + v_0t + D \;\Rightarrow\; s_0 = s(0) = D \;\Rightarrow\; s(t) = \frac{1}{2}at^2 + v_0t + s_0$

65. By Exercise 64 with $a = -9.8$, $s(t) = -4.9t^2 + v_0t + s_0$ and $v(t) = s'(t) = -9.8t + v_0$. So

$[v(t)]^2 = (-9.8t + v_0)^2 = (9.8)^2 t^2 - 19.6v_0t + v_0^2 = v_0^2 + 96.04t^2 - 19.6v_0t = v_0^2 - 19.6\big(-4.9t^2 + v_0t\big)$.

But $-4.9t^2 + v_0t$ is just $s(t)$ without the s_0 term; that is, $s(t) - s_0$. Thus, $[v(t)]^2 = v_0^2 - 19.6\,[s(t) - s_0]$.

66. For the first ball, $s_1(t) = -16t^2 + 48t + 432$ from Example 7. For the second ball, $a(t) = -32 \;\Rightarrow\; v(t) = -32t + C$, but

$v(1) = -32(1) + C = 24 \;\Rightarrow\; C = 56$, so $v(t) = -32t + 56 \;\Rightarrow\; s(t) = -16t^2 + 56t + D$, but

$s(1) = -16(1)^2 + 56(1) + D = 432 \;\Rightarrow\; D = 392$, and $s_2(t) = -16t^2 + 56t + 392$. The balls pass each other

when $s_1(t) = s_2(t) \;\Rightarrow\; -16t^2 + 48t + 432 = -16t^2 + 56t + 392 \;\Leftrightarrow\; 8t = 40 \;\Leftrightarrow\; t = 5$ s.

Another solution: From Exercise 64, we have $s_1(t) = -16t^2 + 48t + 432$ and $s_2(t) = -16t^2 + 24t + 432$.

We now want to solve $s_1(t) = s_2(t-1) \;\Rightarrow\; -16t^2 + 48t + 432 = -16(t-1)^2 + 24(t-1) + 432 \;\Rightarrow$

$48t = 32t - 16 + 24t - 24 \;\Rightarrow\; 40 = 8t \;\Rightarrow\; t = 5$ s.

67. Using Exercise 64 with $a = -32$, $v_0 = 0$, and $s_0 = h$ (the height of the cliff), we know that the height at time t is

$s(t) = -16t^2 + h$. $v(t) = s'(t) = -32t$ and $v(t) = -120 \;\Rightarrow\; -32t = -120 \;\Rightarrow\; t = 3.75$, so

$0 = s(3.75) = -16(3.75)^2 + h \;\Rightarrow\; h = 16(3.75)^2 = 225$ ft.

68. (a) $EIy'' = mg(L-x) + \frac{1}{2}\rho g(L-x)^2 \;\Rightarrow\; EIy' = -\frac{1}{2}mg(L-x)^2 - \frac{1}{6}\rho g(L-x)^3 + C \;\Rightarrow$

$EIy = \frac{1}{6}mg(L-x)^3 + \frac{1}{24}\rho g(L-x)^4 + Cx + D$. Since the left end of the board is fixed, we must have $y = y' = 0$

when $x = 0$. Thus, $0 = -\frac{1}{2}mgL^2 - \frac{1}{6}\rho gL^3 + C$ and $0 = \frac{1}{6}mgL^3 + \frac{1}{24}\rho gL^4 + D$. It follows that

$EIy = \frac{1}{6}mg(L-x)^3 + \frac{1}{24}\rho g(L-x)^4 + \big(\frac{1}{2}mgL^2 + \frac{1}{6}\rho gL^3\big)x - \big(\frac{1}{6}mgL^3 + \frac{1}{24}\rho gL^4\big)$ and

$$f(x) = y = \frac{1}{EI}\left[\tfrac{1}{6}mg(L-x)^3 + \tfrac{1}{24}\rho g(L-x)^4 + \big(\tfrac{1}{2}mgL^2 + \tfrac{1}{6}\rho gL^3\big)x - \big(\tfrac{1}{6}mgL^3 + \tfrac{1}{24}\rho gL^4\big)\right]$$

(b) $f(L) < 0$, so the end of the board is a *distance* approximately $-f(L)$ below the horizontal. From our result in (a), we calculate

$$-f(L) = \frac{-1}{EI}\left[\tfrac{1}{2}mgL^3 + \tfrac{1}{6}\rho gL^4 - \tfrac{1}{6}mgL^3 - \tfrac{1}{24}\rho gL^4\right] = \frac{-1}{EI}\left(\tfrac{1}{3}mgL^3 + \tfrac{1}{8}\rho gL^4\right) = -\frac{gL^3}{EI}\left(\frac{m}{3} + \frac{\rho L}{8}\right)$$

Note: This is positive because g is negative.

69. Marginal cost $= 1.92 - 0.002x = C'(x) \;\Rightarrow\; C(x) = 1.92x - 0.001x^2 + K$. But $C(1) = 1.92 - 0.001 + K = 562 \;\Rightarrow$

$K = 560.081$. Therefore, $C(x) = 1.92x - 0.001x^2 + 560.081 \;\Rightarrow\; C(100) = 742.081$, so the cost of producing 100 items is \$742.08.

70. Let the mass, measured from one end, be $m(x)$. Then $m(0) = 0$ and $\rho = \dfrac{dm}{dx} = x^{-1/2} \;\Rightarrow\; m(x) = 2x^{1/2} + C$ and

$m(0) = C = 0$, so $m(x) = 2\sqrt{x}$. Thus, the mass of the 100-centimeter rod is $m(100) = 2\sqrt{100} = 20$ g.

71. Taking the upward direction to be positive we have that for $0 \le t \le 10$ (using the subscript 1 to refer to $0 \le t \le 10$),

$a_1(t) = -(9 - 0.9t) = v_1'(t) \quad\Rightarrow\quad v_1(t) = -9t + 0.45t^2 + v_0$, but $v_1(0) = v_0 = -10 \quad\Rightarrow$

$v_1(t) = -9t + 0.45t^2 - 10 = s_1'(t) \quad\Rightarrow\quad s_1(t) = -\frac{9}{2}t^2 + 0.15t^3 - 10t + s_0$. But $s_1(0) = 500 = s_0 \quad\Rightarrow$

$s_1(t) = -\frac{9}{2}t^2 + 0.15t^3 - 10t + 500$. $s_1(10) = -450 + 150 - 100 + 500 = 100$, so it takes

more than 10 seconds for the raindrop to fall. Now for $t > 10$, $a(t) = 0 = v'(t) \quad\Rightarrow$

$v(t) = \text{constant} = v_1(10) = -9(10) + 0.45(10)^2 - 10 = -55 \quad\Rightarrow\quad v(t) = -55$.

At 55 m/s, it will take $100/55 \approx 1.8$ s to fall the last 100 m. Hence, the total time is $10 + \frac{100}{55} = \frac{130}{11} \approx 11.8$ s.

72. $v'(t) = a(t) = -22$. The initial velocity is 50 mi/h $= \frac{50 \cdot 5280}{3600} = \frac{220}{3}$ ft/s, so $v(t) = -22t + \frac{220}{3}$.

The car stops when $v(t) = 0 \quad\Leftrightarrow\quad t = \frac{220}{3 \cdot 22} = \frac{10}{3}$. Since $s(t) = -11t^2 + \frac{220}{3}t$, the distance covered is

$s\left(\frac{10}{3}\right) = -11\left(\frac{10}{3}\right)^2 + \frac{220}{3} \cdot \frac{10}{3} = \frac{1100}{9} = 122.\overline{2}$ ft.

73. $a(t) = k$, the initial velocity is 30 mi/h $= 30 \cdot \frac{5280}{3600} = 44$ ft/s, and the final velocity (after 5 seconds) is

50 mi/h $= 50 \cdot \frac{5280}{3600} = \frac{220}{3}$ ft/s. So $v(t) = kt + C$ and $v(0) = 44 \quad\Rightarrow\quad C = 44$. Thus, $v(t) = kt + 44 \quad\Rightarrow$

$v(5) = 5k + 44$. But $v(5) = \frac{220}{3}$, so $5k + 44 = \frac{220}{3} \quad\Rightarrow\quad 5k = \frac{88}{3} \quad\Rightarrow\quad k = \frac{88}{15} \approx 5.87$ ft/s^2.

74. $a(t) = -16 \quad\Rightarrow\quad v(t) = -16t + v_0$ where v_0 is the car's speed (in ft/s) when the brakes were applied. The car stops when

$-16t + v_0 = 0 \quad\Leftrightarrow\quad t = \frac{1}{16}v_0$. Now $s(t) = \frac{1}{2}(-16)t^2 + v_0 t = -8t^2 + v_0 t$. The car travels 200 ft in the time that it takes

to stop, so $s\left(\frac{1}{16}v_0\right) = 200 \quad\Rightarrow\quad 200 = -8\left(\frac{1}{16}v_0\right)^2 + v_0\left(\frac{1}{16}v_0\right) = \frac{1}{32}v_0^2 \quad\Rightarrow\quad v_0^2 = 32 \cdot 200 = 6400 \quad\Rightarrow$

$v_0 = 80$ ft/s $[54.\overline{54}$ mi/h$]$.

75. Let the acceleration be $a(t) = k$ km/h^2. We have $v(0) = 100$ km/h and we can take the initial position $s(0)$ to be 0.

We want the time t_f for which $v(t) = 0$ to satisfy $s(t) < 0.08$ km. In general, $v'(t) = a(t) = k$, so $v(t) = kt + C$, where

$C = v(0) = 100$. Now $s'(t) = v(t) = kt + 100$, so $s(t) = \frac{1}{2}kt^2 + 100t + D$, where $D = s(0) = 0$.

Thus, $s(t) = \frac{1}{2}kt^2 + 100t$. Since $v(t_f) = 0$, we have $kt_f + 100 = 0$ or $t_f = -100/k$, so

$s(t_f) = \dfrac{1}{2}k\left(-\dfrac{100}{k}\right)^2 + 100\left(-\dfrac{100}{k}\right) = 10{,}000\left(\dfrac{1}{2k} - \dfrac{1}{k}\right) = -\dfrac{5{,}000}{k}$. The condition $s(t_f)$ must satisfy is

$-\dfrac{5{,}000}{k} < 0.08 \quad\Rightarrow\quad -\dfrac{5{,}000}{0.08} > k \quad [k \text{ is negative}] \quad\Rightarrow\quad k < -62{,}500$ km/h^2, or equivalently,

$k < -\frac{3125}{648} \approx -4.82$ m/s^2.

76. (a) For $0 \le t \le 3$ we have $a(t) = 60t \quad\Rightarrow\quad v(t) = 30t^2 + C \quad\Rightarrow\quad v(0) = 0 = C \quad\Rightarrow\quad v(t) = 30t^2$, so

$s(t) = 10t^3 + C \quad\Rightarrow\quad s(0) = 0 = C \quad\Rightarrow\quad s(t) = 10t^3$. Note that $v(3) = 270$ and $s(3) = 270$.

For $3 < t \le 17$: $a(t) = -g = -32$ ft/s $\quad\Rightarrow\quad v(t) = -32(t - 3) + C \quad\Rightarrow\quad v(3) = 270 = C \quad\Rightarrow$

$v(t) = -32(t - 3) + 270 \quad\Rightarrow\quad s(t) = -16(t - 3)^2 + 270(t - 3) + C \quad\Rightarrow\quad s(3) = 270 = C \quad\Rightarrow$

$s(t) = -16(t - 3)^2 + 270(t - 3) + 270$. Note that $v(17) = -178$ and $s(17) = 914$.

For $17 < t \le 22$: The velocity increases linearly from -178 ft/s to -18 ft/s during this period, so

$\dfrac{\Delta v}{\Delta t} = \dfrac{-18-(-178)}{22-17} = \dfrac{160}{5} = 32$. Thus, $v(t) = 32(t-17) - 178 \Rightarrow$

$s(t) = 16(t-17)^2 - 178(t-17) + 914$ and $s(22) = 424$ ft.

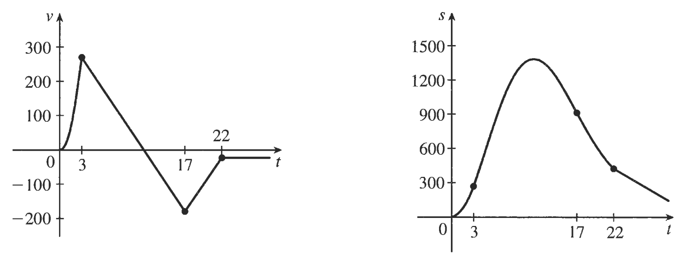
For $t > 22$: $v(t) = -18 \Rightarrow s(t) = -18(t-22) + C$. But $s(22) = 424 = C \Rightarrow s(t) = -18(t-22) + 424$.

Therefore, until the rocket lands, we have

$$v(t) = \begin{cases} 30t^2 & \text{if } 0 \le t \le 3 \\ -32\,(t-3) + 270 & \text{if } 3 < t \le 17 \\ 32(t-17) - 178 & \text{if } 17 < t \le 22 \\ -18 & \text{if } t > 22 \end{cases}$$

and

$$s(t) = \begin{cases} 10t^3 & \text{if } 0 \le t \le 3 \\ -16(t-3)^2 + 270(t-3) + 270 & \text{if } 3 < t \le 17 \\ 16(t-17)^2 - 178\,(t-17) + 914 & \text{if } 17 < t \le 22 \\ -18(t-22) + 424 & \text{if } t > 22 \end{cases}$$



(b) To find the maximum height, set $v(t)$ on $3 < t \le 17$ equal to 0. $-32(t-3) + 270 = 0 \Rightarrow t_1 = 11.4375$ s and the maximum height is $s(t_1) = -16(t_1 - 3)^2 + 270(t_1 - 3) + 270 = 1409.0625$ ft.

(c) To find the time to land, set $s(t) = -18(t-22) + 424 = 0$. Then $t - 22 = \frac{424}{18} = 23.\overline{5}$, so $t \approx 45.6$ s.

77. (a) First note that 90 mi/h $= 90 \times \frac{5280}{3600}$ ft/s $= 132$ ft/s. Then $a(t) = 4\text{ ft/s}^2 \Rightarrow v(t) = 4t + C$, but $v(0) = 0 \Rightarrow C = 0$. Now $4t = 132$ when $t = \frac{132}{4} = 33$ s, so it takes 33 s to reach 132 ft/s. Therefore, taking $s(0) = 0$, we have $s(t) = 2t^2$, $0 \le t \le 33$. So $s(33) = 2178$ ft. 15 minutes $= 15(60) = 900$ s, so for $33 < t \le 933$ we have $v(t) = 132$ ft/s $\Rightarrow s(933) = 132(900) + 2178 = 120{,}978$ ft $= 22.9125$ mi.

(b) As in part (a), the train accelerates for 33 s and travels 2178 ft while doing so. Similarly, it decelerates for 33 s and travels 2178 ft at the end of its trip. During the remaining $900 - 66 = 834$ s it travels at 132 ft/s, so the distance traveled is $132 \cdot 834 = 110{,}088$ ft. Thus, the total distance is $2178 + 110{,}088 + 2178 = 114{,}444$ ft $= 21.675$ mi.

(c) 45 mi $= 45(5280) = 237{,}600$ ft. Subtract $2(2178)$ to take care of the speeding up and slowing down, and we have 233,244 ft at 132 ft/s for a trip of $233{,}244/132 = 1767$ s at 90 mi/h. The total time is $1767 + 2(33) = 1833$ s $= 30$ min 33 s $= 30.55$ min.

(d) $37.5(60) = 2250$ s. $2250 - 2(33) = 2184$ s at maximum speed. $2184(132) + 2(2178) = 292{,}644$ total feet or $292{,}644/5280 = 55.425$ mi.