1. $\frac{d}{dx}\left[\sqrt{x^2+1}+C\right] = \frac{d}{dx}\left[\left(x^2+1\right)^{1/2}+C\right] = \frac{1}{2}\left(x^2+1\right)^{-1/2}\cdot 2x+0 = \frac{x}{\sqrt{x^2+1}}$

2. $\frac{d}{dx}\left[x\sin x+\cos x+C\right] = x\cos x+(\sin x)\cdot 1-\sin x+0 = x\cos x$

3. $\frac{d}{dx}\left[\sin x-\frac{1}{3}\sin^3 x+C\right] = \frac{d}{dx}\left[\sin x-\frac{1}{3}(\sin x)^3+C\right] = \cos x-\frac{1}{3}\cdot 3(\sin x)^2(\cos x)+0$

$= \cos x(1-\sin^2 x) = \cos x(\cos^2 x) = \cos^3 x$

4. $\frac{d}{dx}\left[\frac{2}{3b^2}(bx-2a)\sqrt{a+bx}+C\right]=\frac{d}{dx}\left[\frac{2}{3b^2}(bx-2a)(a+bx)^{1/2}+C\right]$

$= \frac{2}{3b^2}\left[(bx-2a)\cdot\frac{1}{2}(a+bx)^{-1/2}(b)+(a+bx)^{1/2}(b)\right]+0$

$= \frac{2}{3b^2}\cdot\frac{1}{2}b(a+bx)^{-1/2}\left[(bx-2a)+2(a+bx)\right]=\frac{1}{3b\sqrt{a+bx}}\left[3bx\right]=\frac{x}{\sqrt{a+bx}}$

5. $\int(x^2+x^{-2})\,dx=\frac{x^3}{3}+\frac{x^{-1}}{-1}+C=\frac{1}{3}x^3-\frac{1}{x}+C$

6. $\int\left(\sqrt{x^3}+\sqrt[3]{x^2}\right)dx=\int(x^{3/2}+x^{2/3})\,dx=\frac{x^{5/2}}{5/2}+\frac{x^{5/3}}{5/3}+C=\frac{2}{5}x^{5/2}+\frac{3}{5}x^{5/3}+C$

7. $\int\left(x^4-\frac{1}{2}x^3+\frac{1}{4}x-2\right)dx=\frac{x^5}{5}-\frac{1}{2}\frac{x^4}{4}+\frac{1}{4}\frac{x^2}{2}-2x+C=\frac{1}{5}x^5-\frac{1}{8}x^4+\frac{1}{8}x^2-2x+C$

8. $\int(y^3+1.8y^2-2.4y)\,dy=\frac{y^4}{4}+1.8\frac{y^3}{3}-2.4\frac{y^2}{2}+C=\frac{1}{4}y^4+0.6y^3-1.2y^2+C$

9. $\int(1-t)(2+t^2)\,dt=\int(2-2t+t^2-t^3)\,dt=2t-2\frac{t^2}{2}+\frac{t^3}{3}-\frac{t^4}{4}+C=2t-t^2+\frac{1}{3}t^3-\frac{1}{4}t^4+C$

10. $\int v(v^2+2)^2\,dv=\int v(v^4+4v^2+4)\,dv=\int(v^5+4v^3+4v)\,dv=\frac{v^6}{6}+4\frac{v^4}{4}+4\frac{v^2}{2}+C=\frac{1}{6}v^6+v^4+2v^2+C$

11. $\int\frac{x^3-2\sqrt{x}}{x}\,dx=\int\left(\frac{x^3}{x}-\frac{2x^{1/2}}{x}\right)dx=\int(x^2-2x^{-1/2})\,dx=\frac{x^3}{3}-2\frac{x^{1/2}}{1/2}+C=\frac{1}{3}x^3-4\sqrt{x}+C$

12. $\int\left(x^2+1+\frac{1}{x^2+1}\right)dx=\frac{x^3}{3}+x+\tan^{-1}x+C$

13. $\int(\sin x+\sinh x)\,dx=-\cos x+\cosh x+C$

14. $\int(\csc^2 t-2e^t)\,dt=-\cot t-2e^t+C$

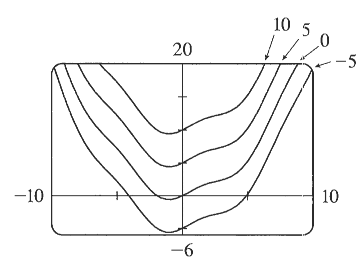
15. $\int(\theta-\csc\theta\,\cot\theta)\,d\theta=\frac{1}{2}\theta^2+\csc\theta+C$

16. $\int\sec t\,(\sec t+\tan t)\,dt=\int(\sec^2 t+\sec t\,\tan t)\,dt=\tan t+\sec t+C$

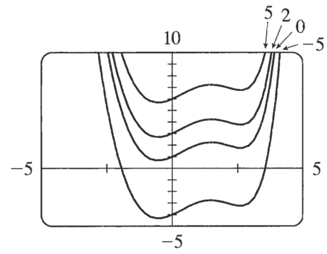
17. $\int(1+\tan^2\alpha)\,d\alpha=\int\sec^2\alpha\,d\alpha=\tan\alpha+C$

18. $\int\frac{\sin 2x}{\sin x}\,dx=\int\frac{2\sin x\,\cos x}{\sin x}\,dx=\int 2\cos x\,dx=2\sin x+C$

19. $\int\left(\cos x+\frac{1}{2}x\right)dx=\sin x+\frac{1}{4}x^2+C$. The members of the family in the figure correspond to $C=-5$, 0, 5, and 10.



20. $\int (e^x - 2x^2)\,dx = e^x - \frac{2}{3}x^3 + C$. The members of the family in the figure correspond to $C = -5, 0, 2$, and 5.



21. $\int_0^2 (6x^2 - 4x + 5)\,dx = \left[6 \cdot \frac{1}{3}x^3 - 4 \cdot \frac{1}{2}x^2 + 5x\right]_0^2 = \left[2x^3 - 2x^2 + 5x\right]_0^2 = (16 - 8 + 10) - 0 = 18$

22. $\int_1^3 (1 + 2x - 4x^3)\,dx = \left[x + 2 \cdot \frac{1}{2}x^2 - 4 \cdot \frac{1}{4}x^4\right]_1^3 = \left[x + x^2 - x^4\right]_1^3 = (3 + 9 - 81) - (1 + 1 - 1) = -69 - 1 = -70$

23. $\int_{-1}^0 (2x - e^x)\,dx = \left[x^2 - e^x\right]_{-1}^0 = (0 - 1) - \left(1 - e^{-1}\right) = -2 + 1/e$

24. $\int_{-2}^0 (u^5 - u^3 + u^2)\,du = \left[\frac{1}{6}u^6 - \frac{1}{4}u^4 + \frac{1}{3}u^3\right]_{-2}^0 = 0 - \left(\frac{32}{3} - 4 - \frac{8}{3}\right) = -4$

25. $\int_{-2}^2 (3u + 1)^2\,du = \int_{-2}^2 \left(9u^2 + 6u + 1\right)\,du = \left[9 \cdot \frac{1}{3}u^3 + 6 \cdot \frac{1}{2}u^2 + u\right]_{-2}^2 = \left[3u^3 + 3u^2 + u\right]_{-2}^2$
$= (24 + 12 + 2) - (-24 + 12 - 2) = 38 - (-14) = 52$

26. $\int_0^4 (2v + 5)(3v - 1)\,dv = \int_0^4 (6v^2 + 13v - 5)\,dv = \left[6 \cdot \frac{1}{3}v^3 + 13 \cdot \frac{1}{2}v^2 - 5v\right]_0^4 = \left[2v^3 + \frac{13}{2}v^2 - 5v\right]_0^4$
$= (128 + 104 - 20) - 0 = 212$

27. $\int_1^4 \sqrt{t}\,(1 + t)\,dt = \int_1^4 (t^{1/2} + t^{3/2})\,dt = \left[\frac{2}{3}t^{3/2} + \frac{2}{5}t^{5/2}\right]_1^4 = \left(\frac{16}{3} + \frac{64}{5}\right) - \left(\frac{2}{3} + \frac{2}{5}\right) = \frac{14}{3} + \frac{62}{5} = \frac{256}{15}$

28. $\int_0^9 \sqrt{2t}\,dt = \int_0^9 \sqrt{2}\,t^{1/2}\,dt = \left[\sqrt{2} \cdot \frac{2}{3}t^{3/2}\right]_0^9 = \sqrt{2} \cdot \frac{2}{3} \cdot 27 - 0 = 18\sqrt{2}$

29. $\displaystyle\int_{-2}^{-1} \left(4y^3 + \frac{2}{y^3}\right) dy = \left[4 \cdot \frac{1}{4}y^4 + 2 \cdot \frac{1}{-2}y^{-2}\right]_{-2}^{-1} = \left[y^4 - \frac{1}{y^2}\right]_{-2}^{-1} = (1 - 1) - \left(16 - \frac{1}{4}\right) = -\frac{63}{4}$

30. $\displaystyle\int_1^2 \frac{y + 5y^7}{y^3}\,dy = \int_1^2 (y^{-2} + 5y^4)\,dy = \left[-y^{-1} + 5 \cdot \frac{1}{5}y^5\right]_1^2 = \left[-\frac{1}{y} + y^5\right]_1^2 = \left(-\frac{1}{2} + 32\right) - (-1 + 1) = \frac{63}{2}$

31. $\int_0^1 x\left(\sqrt[3]{x} + \sqrt[4]{x}\right)\,dx = \int_0^1 (x^{4/3} + x^{5/4})\,dx = \left[\frac{3}{7}x^{7/3} + \frac{4}{9}x^{9/4}\right]_0^1 = \left(\frac{3}{7} + \frac{4}{9}\right) - 0 = \frac{55}{63}$

32. $\int_0^5 (2e^x + 4\cos x)\,dx = \left[2e^x + 4\sin x\right]_0^5 = (2e^5 + 4\sin 5) - (2e^0 + 4\sin 0) = 2e^5 + 4\sin 5 - 2 \approx 290.99$

33. $\int_1^4 \sqrt{5/x}\,dx = \sqrt{5}\int_1^4 x^{-1/2}\,dx = \sqrt{5}\left[2\sqrt{x}\right]_1^4 = \sqrt{5}\,(2 \cdot 2 - 2 \cdot 1) = 2\sqrt{5}$

34. $\displaystyle\int_1^9 \frac{3x - 2}{\sqrt{x}}\,dx = \int_1^9 (3x^{1/2} - 2x^{-1/2})\,dx = \left[3 \cdot \frac{2}{3}x^{3/2} - 2 \cdot 2x^{1/2}\right]_1^9 = \left[2x^{3/2} - 4x^{1/2}\right]_1^9$
$= (54 - 12) - (2 - 4) = 44$

35. $\int_0^\pi (4\sin\theta - 3\cos\theta)\,d\theta = \left[-4\cos\theta - 3\sin\theta\right]_0^\pi = (4 - 0) - (-4 - 0) = 8$

36. $\int_{\pi/4}^{\pi/3} \sec\theta\tan\theta\,d\theta = \left[\sec\theta\right]_{\pi/4}^{\pi/3} = \sec\frac{\pi}{3} - \sec\frac{\pi}{4} = 2-\sqrt{2}$

37. $\displaystyle\int_0^{\pi/4} \frac{1+\cos^2\theta}{\cos^2\theta}\,d\theta = \int_0^{\pi/4}\left(\frac{1}{\cos^2\theta}+\frac{\cos^2\theta}{\cos^2\theta}\right)d\theta = \int_0^{\pi/4}(\sec^2\theta+1)\,d\theta$

$= \left[\tan\theta+\theta\right]_0^{\pi/4} = \left(\tan\frac{\pi}{4}+\frac{\pi}{4}\right)-(0+0) = 1+\frac{\pi}{4}$

38. $\displaystyle\int_0^{\pi/3} \frac{\sin\theta+\sin\theta\tan^2\theta}{\sec^2\theta}\,d\theta = \int_0^{\pi/3}\frac{\sin\theta\,(1+\tan^2\theta)}{\sec^2\theta}\,d\theta = \int_0^{\pi/3}\frac{\sin\theta\sec^2\theta}{\sec^2\theta}\,d\theta = \int_0^{\pi/3}\sin\theta\,d\theta$

$= \left[-\cos\theta\right]_0^{\pi/3} = -\frac{1}{2}-(-1) = \frac{1}{2}$

39. $\displaystyle\int_1^{64}\frac{1+\sqrt[3]{x}}{\sqrt{x}}\,dx = \int_1^{64}\left(\frac{1}{x^{1/2}}+\frac{x^{1/3}}{x^{1/2}}\right)dx = \int_1^{64}\left(x^{-1/2}+x^{(1/3)-(1/2)}\right)dx = \int_1^{64}(x^{-1/2}+x^{-1/6})\,dx$

$= \left[2x^{1/2}+\frac{6}{5}x^{5/6}\right]_1^{64} = \left(16+\frac{192}{5}\right)-\left(2+\frac{6}{5}\right) = 14+\frac{186}{5} = \frac{256}{5}$

40. $\displaystyle\int_{-10}^{10}\frac{2e^x}{\sinh x+\cosh x}\,dx = \int_{-10}^{10}\frac{2e^x}{\frac{e^x-e^{-x}}{2}+\frac{e^x+e^{-x}}{2}}\,dx = \int_{-10}^{10}\frac{2e^x}{e^x}\,dx = \int_{-10}^{10}2\,dx = \left[2x\right]_{-10}^{10} = 20-(-20) = 40$

41. $\displaystyle\int_0^{1/\sqrt{3}}\frac{t^2-1}{t^4-1}\,dt = \int_0^{1/\sqrt{3}}\frac{t^2-1}{(t^2+1)(t^2-1)}\,dt = \int_0^{1/\sqrt{3}}\frac{1}{t^2+1}\,dt = \left[\arctan t\right]_0^{1/\sqrt{3}} = \arctan\left(1/\sqrt{3}\right)-\arctan 0$

$= \frac{\pi}{6}-0 = \frac{\pi}{6}$

42. $\displaystyle\int_1^2\frac{(x-1)^3}{x^2}\,dx = \int_1^2\frac{x^3-3x^2+3x-1}{x^2}\,dx = \int_1^2\left(x-3+\frac{3}{x}-\frac{1}{x^2}\right)dx = \left[\frac{1}{2}x^2-3x+3\ln|x|+\frac{1}{x}\right]_1^2$

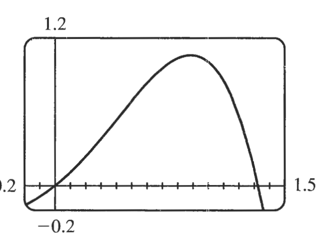
$= \left(2-6+3\ln 2+\frac{1}{2}\right)-\left(\frac{1}{2}-3+0+1\right) = 3\ln 2-2$

43. $\int_{-1}^2(x-2|x|)\,dx = \int_{-1}^0[x-2(-x)]\,dx+\int_0^2[x-2(x)]\,dx = \int_{-1}^0 3x\,dx+\int_0^2(-x)\,dx = 3\left[\frac{1}{2}x^2\right]_{-1}^0-\left[\frac{1}{2}x^2\right]_0^2$

$= 3\left(0-\frac{1}{2}\right)-(2-0) = -\frac{7}{2} = -3.5$

44. $\int_0^{3\pi/2}|\sin x|\,dx = \int_0^\pi\sin x\,dx+\int_\pi^{3\pi/2}(-\sin x)\,dx = \left[-\cos x\right]_0^\pi+\left[\cos x\right]_\pi^{3\pi/2} = [1-(-1)]+[0-(-1)] = 2+1 = 3$

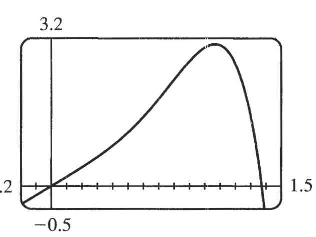
45. The graph shows that $y = x+x^2-x^4$ has x-intercepts at $x=0$ and at $x=a\approx 1.32$. So the area of the region that lies under the curve and above the x-axis is



$\int_0^a(x+x^2-x^4)\,dx = \left[\frac{1}{2}x^2+\frac{1}{3}x^3-\frac{1}{5}x^5\right]_0^a$

$= \left(\frac{1}{2}a^2+\frac{1}{3}a^3-\frac{1}{5}a^5\right)-0\approx 0.84$

46. The graph shows that $y = 2x+3x^4-2x^6$ has x-intercepts at $x=0$ and at $x=a\approx 1.37$. So the area of the region that lies under the curve and above the x-axis is



$\int_0^a(2x+3x^4-2x^6)\,dx = \left[x^2+\frac{3}{5}x^5-\frac{2}{7}x^7\right]_0^a$

$= \left(a^2+\frac{3}{5}a^5-\frac{2}{7}a^7\right)-0\approx 2.18$

47. $A = \int_0^2 (2y - y^2)\,dy = \left[y^2 - \frac{1}{3}y^3\right]_0^2 = \left(4 - \frac{8}{3}\right) - 0 = \frac{4}{3}$

48. $y = \sqrt[4]{x} \;\Rightarrow\; x = y^4$, so $A = \int_0^1 y^4\,dy = \left[\frac{1}{5}y^5\right]_0^1 = \frac{1}{5}$.

49. If $w'(t)$ is the rate of change of weight in pounds per year, then $w(t)$ represents the weight in pounds of the child at age t. We know from the Net Change Theorem that $\int_5^{10} w'(t)\,dt = w(10) - w(5)$, so the integral represents the increase in the child's weight (in pounds) between the ages of 5 and 10.

50. $\int_a^b I(t)\,dt = \int_a^b Q'(t)\,dt = Q(b) - Q(a)$ by the Net Change Theorem, so it represents the change in the charge Q from time $t = a$ to $t = b$.

51. Since $r(t)$ is the rate at which oil leaks, we can write $r(t) = -V'(t)$, where $V(t)$ is the volume of oil at time t. [Note that the minus sign is needed because V is decreasing, so $V'(t)$ is negative, but $r(t)$ is positive.] Thus, by the Net Change Theorem, $\int_0^{120} r(t)\,dt = -\int_0^{120} V'(t)\,dt = -\left[V(120) - V(0)\right] = V(0) - V(120)$, which is the number of gallons of oil that leaked from the tank in the first two hours (120 minutes).

52. By the Net Change Theorem, $\int_0^{15} n'(t)\,dt = n(15) - n(0) = n(15) - 100$ represents the increase in the bee population in 15 weeks. So $100 + \int_0^{15} n'(t)\,dt = n(15)$ represents the total bee population after 15 weeks.

53. By the Net Change Theorem, $\int_{1000}^{5000} R'(x)\,dx = R(5000) - R(1000)$, so it represents the increase in revenue when production is increased from 1000 units to 5000 units.

54. The slope of the trail is the rate of change of the elevation E, so $f(x) = E'(x)$. By the Net Change Theorem, $\int_3^5 f(x)\,dx = \int_3^5 E'(x)\,dx = E(5) - E(3)$ is the change in the elevation E between $x = 3$ miles and $x = 5$ miles from the start of the trail.

55. In general, the unit of measurement for $\int_a^b f(x)\,dx$ is the product of the unit for $f(x)$ and the unit for x. Since $f(x)$ is measured in newtons and x is measured in meters, the units for $\int_0^{100} f(x)\,dx$ are newton-meters. (A newton-meter is abbreviated N·m and is called a joule.)

56. The units for $a(x)$ are pounds per foot and the units for x are feet, so the units for da/dx are pounds per foot per foot, denoted $(\text{lb/ft})/\text{ft}$. The unit of measurement for $\int_2^8 a(x)\,dx$ is the product of pounds per foot and feet; that is, pounds.

57. (a) Displacement $= \int_0^3 (3t - 5)\,dt = \left[\frac{3}{2}t^2 - 5t\right]_0^3 = \frac{27}{2} - 15 = -\frac{3}{2}$ m

(b) Distance traveled $= \int_0^3 |3t - 5|\,dt = \int_0^{5/3} (5 - 3t)\,dt + \int_{5/3}^3 (3t - 5)\,dt$

$$= \left[5t - \tfrac{3}{2}t^2\right]_0^{5/3} + \left[\tfrac{3}{2}t^2 - 5t\right]_{5/3}^3 = \tfrac{25}{3} - \tfrac{3}{2}\cdot\tfrac{25}{9} + \tfrac{27}{2} - 15 - \left(\tfrac{3}{2}\cdot\tfrac{25}{9} - \tfrac{25}{3}\right) = \tfrac{41}{6}\text{ m}$$

58. (a) Displacement $= \int_1^6 (t^2 - 2t - 8)\,dt = \left[\frac{1}{3}t^3 - t^2 - 8t\right]_1^6 = (72 - 36 - 48) - \left(\frac{1}{3} - 1 - 8\right) = -\frac{10}{3}$ m

(b) Distance traveled $= \int_1^6 |t^2 - 2t - 8|\,dt = \int_1^6 |(t - 4)(t + 2)|\,dt$

$$= \int_1^4 (-t^2 + 2t + 8)\,dt + \int_4^6 (t^2 - 2t - 8)\,dt = \left[-\tfrac{1}{3}t^3 + t^2 + 8t\right]_1^4 + \left[\tfrac{1}{3}t^3 - t^2 - 8t\right]_4^6$$

$$= \left(-\tfrac{64}{3} + 16 + 32\right) - \left(-\tfrac{1}{3} + 1 + 8\right) + (72 - 36 - 48) - \left(\tfrac{64}{3} - 16 - 32\right) = \tfrac{98}{3}\text{ m}$$

59. (a) $v'(t) = a(t) = t + 4 \;\Rightarrow\; v(t) = \frac{1}{2}t^2 + 4t + C \;\Rightarrow\; v(0) = C = 5 \;\Rightarrow\; v(t) = \frac{1}{2}t^2 + 4t + 5$ m/s

(b) Distance traveled $= \int_0^{10} |v(t)|\,dt = \int_0^{10} \left|\frac{1}{2}t^2 + 4t + 5\right| dt = \int_0^{10} \left(\frac{1}{2}t^2 + 4t + 5\right) dt = \left[\frac{1}{6}t^3 + 2t^2 + 5t\right]_0^{10}$

$= \frac{500}{3} + 200 + 50 = 416\frac{2}{3}$ m

60. (a) $v'(t) = a(t) = 2t + 3 \;\Rightarrow\; v(t) = t^2 + 3t + C \;\Rightarrow\; v(0) = C = -4 \;\Rightarrow\; v(t) = t^2 + 3t - 4$

(b) Distance traveled $= \int_0^3 \left|t^2 + 3t - 4\right| dt = \int_0^3 |(t+4)(t-1)|\,dt = \int_0^1 \left(-t^2 - 3t + 4\right) dt + \int_1^3 \left(t^2 + 3t - 4\right) dt$

$= \left[-\frac{1}{3}t^3 - \frac{3}{2}t^2 + 4t\right]_0^1 + \left[\frac{1}{3}t^3 + \frac{3}{2}t^2 - 4t\right]_1^3$

$= \left(-\frac{1}{3} - \frac{3}{2} + 4\right) + \left(9 + \frac{27}{2} - 12\right) - \left(\frac{1}{3} + \frac{3}{2} - 4\right) = \frac{89}{6}$ m

61. Since $m'(x) = \rho(x)$, $m = \int_0^4 \rho(x)\,dx = \int_0^4 \left(9 + 2\sqrt{x}\right) dx = \left[9x + \frac{4}{3}x^{3/2}\right]_0^4 = 36 + \frac{32}{3} - 0 = \frac{140}{3} = 46\frac{2}{3}$ kg.

62. By the Net Change Theorem, the amount of water that flows from the tank during the first 10 minutes is

$\int_0^{10} r(t)\,dt = \int_0^{10} (200 - 4t)\,dt = \left[200t - 2t^2\right]_0^{10} = (2000 - 200) - 0 = 1800$ liters.

63. Let s be the position of the car. We know from Equation 2 that $s(100) - s(0) = \int_0^{100} v(t)\,dt$. We use the Midpoint Rule for $0 \le t \le 100$ with $n = 5$. Note that the length of each of the five time intervals is 20 seconds $= \frac{20}{3600}$ hour $= \frac{1}{180}$ hour. So the distance traveled is

$\int_0^{100} v(t)\,dt \approx \frac{1}{180}[v(10) + v(30) + v(50) + v(70) + v(90)] = \frac{1}{180}(38 + 58 + 51 + 53 + 47) = \frac{247}{180} \approx 1.4$ miles.

64. (a) By the Net Change Theorem, the total amount spewed into the atmosphere is $Q(6) - Q(0) = \int_0^6 r(t)\,dt = Q(6)$ since $Q(0) = 0$. The rate $r(t)$ is positive, so Q is an increasing function. Thus, an upper estimate for $Q(6)$ is R_6 and a lower estimate for $Q(6)$ is L_6. $\Delta t = \dfrac{b-a}{n} = \dfrac{6-0}{6} = 1$.

$R_6 = \sum_{i=1}^{6} r(t_i)\,\Delta t = 10 + 24 + 36 + 46 + 54 + 60 = 230$ tonnes.

$L_6 = \sum_{i=1}^{6} r(t_{i-1})\,\Delta t = R_6 + r(0) - r(6) = 230 + 2 - 60 = 172$ tonnes.

(b) $\Delta t = \dfrac{b-a}{n} = \dfrac{6-0}{3} = 2$. $Q(6) \approx M_3 = 2[r(1) + r(3) + r(5)] = 2(10 + 36 + 54) = 2(100) = 200$ tonnes.

65. From the Net Change Theorem, the increase in cost if the production level is raised from 2000 yards to 4000 yards is $C(4000) - C(2000) = \int_{2000}^{4000} C'(x)\,dx$.

$\int_{2000}^{4000} C'(x)\,dx = \int_{2000}^{4000} \left(3 - 0.01x + 0.000006x^2\right) dx = \left[3x - 0.005x^2 + 0.000002x^3\right]_{2000}^{4000} = 60{,}000 - 2{,}000 = \$58{,}000$

66. By the Net Change Theorem, the amount of water after four days is

$$25{,}000 + \int_0^4 r(t)\,dt \approx 25{,}000 + M_4 = 25{,}000 + \frac{4-0}{4}[r(0.5) + r(1.5) + r(2.5) + r(3.5)]$$

$$\approx 25{,}000 + [1500 + 1770 + 740 + (-690)] = 28{,}320 \text{ liters}$$

67. (a) We can find the area between the Lorenz curve and the line $y = x$ by subtracting the area under $y = L(x)$ from the area under $y = x$. Thus,

$$\text{coefficient of inequality} = \frac{\text{area between Lorenz curve and line } y = x}{\text{area under line } y = x} = \frac{\int_0^1 [x - L(x)]\,dx}{\int_0^1 x\,dx}$$

$$= \frac{\int_0^1 [x - L(x)]\,dx}{[x^2/2]_0^1} = \frac{\int_0^1 [x - L(x)]\,dx}{1/2} = 2\int_0^1 [x - L(x)]\,dx$$

(b) $L(x) = \frac{5}{12}x^2 + \frac{7}{12}x \quad\Rightarrow\quad L(50\%) = L\left(\frac{1}{2}\right) = \frac{5}{48} + \frac{7}{24} = \frac{19}{48} = 0.3958\overline{3}$, so the bottom 50% of the households receive at most about 40% of the income. Using the result in part (a),

$$\text{coefficient of inequality} = 2\int_0^1 [x - L(x)]\,dx = 2\int_0^1 \left(x - \tfrac{5}{12}x^2 - \tfrac{7}{12}x\right)dx = 2\int_0^1 \left(\tfrac{5}{12}x - \tfrac{5}{12}x^2\right)dx$$

$$= 2\int_0^1 \tfrac{5}{12}\left(x - x^2\right)dx = \tfrac{5}{6}\left[\tfrac{1}{2}x^2 - \tfrac{1}{3}x^3\right]_0^1 = \tfrac{5}{6}\left(\tfrac{1}{2} - \tfrac{1}{3}\right) = \tfrac{5}{6}\left(\tfrac{1}{6}\right) = \tfrac{5}{36}$$

68. (a) From Exercise 4.1.72(a), $v(t) = 0.00146t^3 - 0.11553t^2 + 24.98169t - 21.26872$.

(b) $h(125) - h(0) = \int_0^{125} v(t)\,dt = \left[0.000365t^4 - 0.03851t^3 + 12.490845t^2 - 21.26872t\right]_0^{125} \approx 206{,}407$ ft